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Block	Italic	Transliteration	Block	Italic	Transliteration
А а	*А а*	A, a	Р р	*Р р*	R, r
Б б	*Б б*	B, b	С с	*С с*	S, s
В в	*В в*	V, v	Т т	*Т т*	T, t
Г г	*Г г*	G, g	У у	*У у*	U, u
Д д	*Д д*	D, d	Ф ф	*Ф ф*	F, f
Е е	*Е е*	Ye, ye; E, e*	Х х	*Х х*	Kh, kh
Ж ж	*Ж ж*	Zh, zh	Ц ц	*Ц ц*	Ts, ts
З з	*З з*	Z, z	Ч ч	*Ч ч*	Ch, ch
И и	*И и*	I, i	Ш ш	*Ш ш*	Sh, sh
Й й	*Й й*	Y, y	Щ щ	*Щ щ*	Shch, shch
К к	*К к*	K, k	Ъ ъ	*Ъ ъ*	"
Л л	*Л л*	L, l	Ы ы	*Ы ы*	Y, y
М м	*М м*	M, m	Ь ь	*Ь ь*	'
Н н	*Н н*	N, n	Э э	*Э э*	E, e
О о	*О о*	O, o	Ю ю	*Ю ю*	Yu, yu
П п	*П п*	P, p	Я я	*Я я*	Ya, ya

*ye initially, after vowels, and after ъ, ь; e elsewhere. When written as ё in Russian, transliterate as yё or ё.

RUSSIAN AND ENGLISH TRIGONOMETRIC FUNCTIONS

Russian	English	Russian	English	Russian	English
sin	sin	sh	sinh	arc sh	$\sinh^{-1}$
cos	cos	ch	cosh	arc ch	$\cosh^{-1}$
tg	tan	th	tanh	arc th	$\tanh^{-1}$
ctg	cot	cth	coth	arc cth	$\coth^{-1}$
sec	sec	sch	sech	arc sch	sech^{-1}
cosec	csc	csch	csch	arc csch	csch^{-1}

Russian	English
rot	curl
lg	log

APPLICATION OF THE METHOD OF HARMONIC LINEARIZATION FOR INVESTIGATING SLIP MODES IN A DIGITAL TRACKING SYSTEM WITH VARIABLE STRUCTURE

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At the present there are not any accurate methods for solving a nonlinear differential equation of any order. Therefore, used very frequently are approximation methods to study nonlinear control systems, which allow obtaining results with an accuracy sufficient for practice. In the proposed work the problem is stated concerning the application of the widely known method of harmonic linearization of Ye.P. Popov for investigating slip methods in a system with variable structure (SPS).

A very important characteristic of the SPS's is the fact that they allow ensuring a high-quality control of the free movement of objects with constant parameters, and in the case of a change in the parameters of an object over a wide range they allow constructing a system which is barely sensitive to these changes.

In reference [1] a detailed description is given of the principle of construction of the SPS, and methods of the phase plane are widely used. Problems of harmonic linearization and frequency methods of the investigation of the SPS are examined in references [2], [3] and [4].

Let us examine the digital tracking system with variable structure (TsSS SPS) in the control of pit-shaft hoisting machines along a track. Let us take the closed and opened structures as the initial structures. The component (Fig. 1), connected to the input of the

control system, serves as the element for switching of the structure. The logical law of switching of the structure has the form

$$F(x, px) = \begin{cases} x \text{ when } x\frac{dx}{dt} > 0; \\ 0 \text{ when } x\frac{dx}{dt} < 0, \end{cases} \tag{1}$$

where F(x, px) is the nonlinear function; x - the output value.

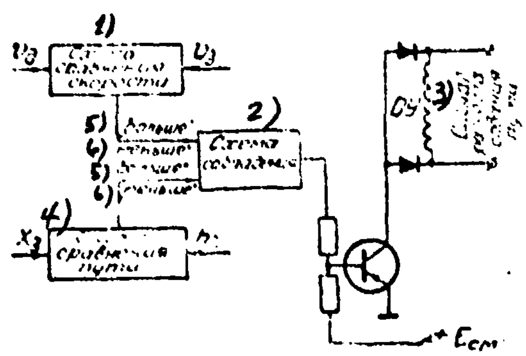

Fig. 1. Block diagram of the switching element: $v_д$, $v_з$ - acting and assigned rates of movment; $h_з$, $x_з$ - traversed and assigned paths of movement. Key: 1) Rate comparison circuit; 2) Coincidence circuit; 3) Error signal of track; 4) Comparison circuit of track*; 5) "greater"; 6) "less." [Translator's note:*track and path are used for the same word in the Russian.]

Since the element for switching of the structure is connected at the output of the digital-analog (Ts-A) converter of the register of the matching of the track [path], then the statistical characteristic and also the input and output signals of the combined nonlinear element (element of switching of the structure + converter Ts-A), in the presence of a constant and slowly changing shift in x^{0}, will have the form depicted on Fig. 2. In this case, with the expansion of the nonlinear function F(x, px) in Fourier series, there appears the slowly changing constant component, owing to which the harmonic linearization of nonlinearity takes such a form [5]

$$F(x, px) = F^{0}(x^{\circ}, \bar{A}, \omega, \alpha) + a(x^{\circ}, \bar{A}, \omega, \alpha)x^{*} + \frac{b(\bar{A}, x^{\circ}, \omega, \alpha)}{\omega}px^{*} + \tag{2}$$

higher harmonics,

where

$$F^{0} = \frac{1}{2\pi}\int_{0}^{2\pi} F(x^{\circ} + \bar{A}\sin\varphi, \bar{A}\omega\cos\varphi)\,d\varphi; \tag{3}$$

$$a = \frac{1}{\pi \bar{A}}\int_0^{2\pi} F(x^* + \bar{A}\sin\varphi,\ \bar{A}\omega\cos\varphi)\sin\varphi\, d\varphi; \qquad (4)$$

$$b = \frac{1}{\pi \bar{A}}\int_0^{2\pi} F(x^* + \bar{A}\sin\varphi,\ \bar{A}\omega\cos\varphi)\cos\varphi\, d\varphi, \qquad (5)$$

x^* is the self-oscillating component with a slowly changing amplitude $\bar{A}$ and frequency ω ; $\alpha = \arcsin\frac{B}{A}$ - switching angle of the structure; $\bar{A} = \frac{A}{\Delta S}$; A - amplitude of the input signal; S - quantization increment.

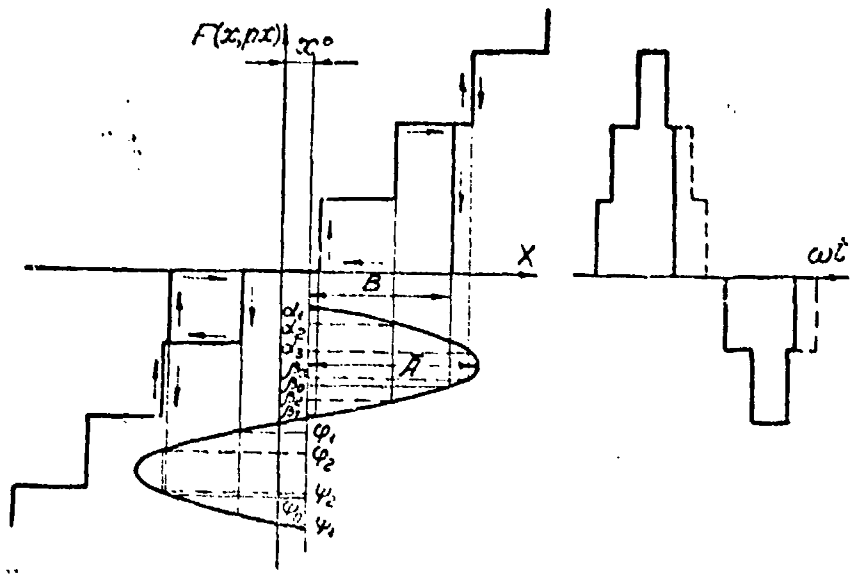

Fig. 2. Statistical characteristic and nature of the input and output signals in the SPS with the Ts-A converter.

If we consider that at the moments of dimensionless time α_i and φ_i the output function F(x, px) by a jump increases by one, and at moments β_i and ψ_i by a jump is decreased by one, and at other time values it remains constant and with a change in the angle from β_0 to π and from ψ_0 to 2π is equal to 0, then the coefficients of harmonic linearization F^0, a and b, computed from formulas (3), (4) and (5), have the form

$$F^\circ = \frac{1}{\pi}\sum_{i=1}^{K}\left(\arcsin\frac{\frac{2i-1}{2}+x^\circ}{\bar{A}} - \arcsin\frac{\frac{2i-1}{2}-x^\circ}{\bar{A}}\right); \qquad (6)$$

$$a = \frac{2}{\pi \bar{A}^2} \sum_{i=1}^{k} \left[\sqrt{\bar{A}^2 - \left(\frac{2i-1}{2} - x^\circ\right)^2} \right.$$

$$\left. + \sqrt{\bar{A}^2 - \left(\frac{2i-1}{2} + x^\circ\right)^2} \right] - \frac{1}{\pi}\left(\alpha - \frac{\sin 2\alpha}{2}\right); \tag{7}$$

$$b = \frac{1}{\pi}\sin^2\alpha, \tag{8}$$

where i is the integer of steps of the characteristic; k - maximal number of steps of the characteristic covered by the input signal.

For the TsSS of the shaft-pit hoist hoisting machine, the multi-step characteristic can be reduced to a relay by applying the variable quantization increment [6]. By keeping this condition in mind and knowing that in the slip mode the control action of the relay element is a sequence of pulse of one sign [7], let us write conditions of the existence of the slip mode for one quantization increment.

Since the condition of operation in one direction has the form

$$\bar{A} + 0{,}5 > |x^\circ| > 0{,}5 - x^\circ, \tag{9}$$

and the condition of the absence of operation in the other direction and in the direction of the change in the quantum is

$$\left.\begin{array}{ll} |x^\circ| + 0{,}5 > \bar{A}, & \text{if } |x^\circ| < 0{,}5, \\ 1{,}5 - |x^\circ| > \bar{A}, & \text{if } 0{,}5 \leqslant |x^\circ| < 1{,}0, \end{array}\right\} \tag{10}$$

then the condition of the existence of the slip mode denotes the combined fulfillment of conditions (9) and (10).

Taking conditions (9) and (10) into account, coefficients of harmonic linearization for the relay characteristic ($i = 1$) in the slip mode will take the form

$$F^\circ = \frac{1}{2\pi}\left[\left(\pi - 2\arcsin\frac{0{,}5 - x^\circ}{\bar{A}}\right) - \bar{A}(1 - \cos\alpha)\right]; \tag{11}$$

$$a = \frac{2}{\pi \bar{A}^2}\sqrt{\bar{A}^2 - (0{,}5 - x^\circ)^2} - \frac{1}{2\pi}\left(\alpha - \frac{\sin 2\alpha}{2}\right); \tag{12}$$

$$b = \frac{1}{2\pi}\sin^2\alpha. \tag{13}$$

By representing the slip movement as a slow change in the coordinate and the pulsations imposed on it, we will distinguish the slowly changing and periodic components. Then instead of one harmonically linearized equation, we will obtain two kinds, respectively, for the

slowly changing and self-oscillating components of the slip movement [5]:

$$\begin{aligned} &L(p)x^{\circ} + N(p)F^{\circ} = S(p)f(t); \\ &L(p)x^{*} + N(p)\left(a + \frac{b}{\omega}p\right)x^{*} = 0, \end{aligned} \qquad (14)$$

where L(p), N(p) and S(p) are polynomials, and f(t) is the external disturbance.

To find the periodic solution, let us examine the second equation of system (14). The differential equation of the TsSS SPS by the pit-shaft hoist for the self-oscillating component will have the form

$$\left[p^5 + A_1p^4 + A_2p^3 + A_3p^2 + A_4p + A_5\left(a + \frac{b}{\omega}p\right)\right]x^{*} = 0, \qquad (15)$$

where coefficients

$$A_1 = a_1; \quad A_2 = 0,4a_1^2; \quad A_3 = 0,08a_1^3; \quad A_4 = 0,008a_1^4$$

satisfy the criterion of optimal figure of merit [6]; $a_1 = \frac{y_1}{\tau_p}$; y_1 – the time factor; τ_p – calculated duration of the transient process.

Having substituted p = $j\omega$ into equation (15), let us select the real and imaginary parts and equate them to zero

$$\left.\begin{aligned} &X(\bar{A}, x^{\circ}, \omega, \alpha) = A_1\omega^4 - A_3\omega^2 + A_5a(\bar{A}, x^{\circ}, \omega, \alpha) = 0; \\ &Y(\bar{A}, x^{\circ}, \omega, \alpha) = \omega^5 - A_2\omega^3 + \left[A_4 + A_5\frac{b(\bar{A}, \omega, x^{\circ}, \alpha)}{\omega}\right]\omega = 0. \end{aligned}\right\} \qquad (16)$$

We will solve the obtained system of equations grapho-analytically. We define the amplification factor A_5 from the the first equation of system (16)

$$A_5 = \frac{\omega^2(A_3 - A_1\omega^2)}{a(\bar{A}, x^{\circ}, \omega, \alpha)}. \qquad (17)$$

To find the amplitude of pulsations $\bar{A}$ and frequency ω, let us use the following method. From the system of equations (16), we find

$$d = \frac{b(\bar{A}, x^{\circ}, \omega, \alpha)}{a(\bar{A}, x, \omega, \alpha)} = \frac{A_2\omega^2 - \omega^4 - A_4}{\omega(A_3 - A_1\omega^2)}. \qquad (18)$$

Equation (18) can be solved graphically (Fig. 3). To do this, from equations (12) and (13) we construct in the second quadrant dependences of quantity d on the amplitude of pulsations $\bar{A}$ for different shifts in x° when α = const. From equation (12) we plot the dependence of the coefficient of harmonic linearization a on the

amplitude of pulsations $\bar{A}$ for different values of shifts in x^o at the same value of α, and from equation (18) we plot the cependence of the ratio of d on the frequency of pulsations $\Omega = \frac{\omega}{\omega_1}$. Being assigned values of ω_i, it is possible for each taken values of α and x^o to determine the amplitude of pulsations $\bar{A}$ and coefficient of harmonic linearization a_i, and then from formula (17) values of the necessary amplification factors of the TsSS SPS in the slip mode are found. The sequence of the determination of $\bar{A}$ and a_i is shown by arrows on Fig. 3, and the dependence of the amplification factor on the amplitude of pulsations, found by the grapho-analytical method described above, is given on Fig. 4.

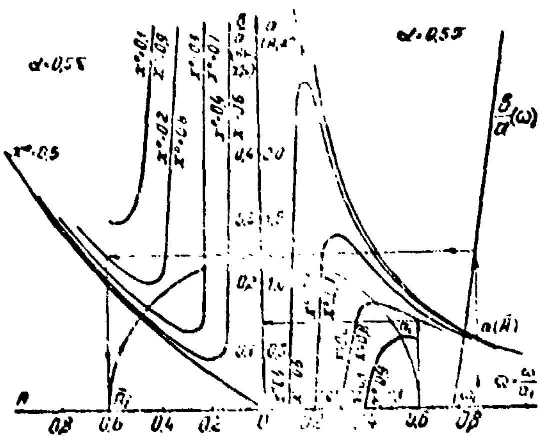

Fig. 3. Sequence of determination of ($\bar{A}$, x^o, ω, α) according to the assigned ω, α and x^o.

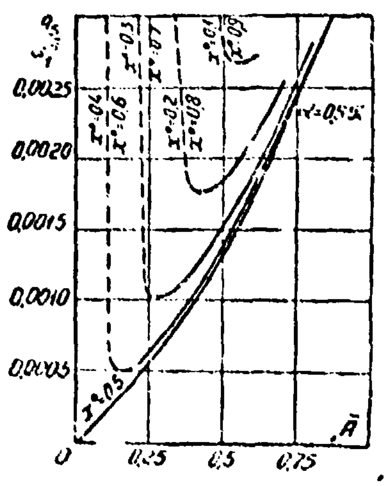

Fig. 4. Dependence of the amplification factor of the TsSS SPS on the amplitude of pulsations in the slip mode: - - - - - unstable part of the characteristic; ———— - stable part of the characteristic.

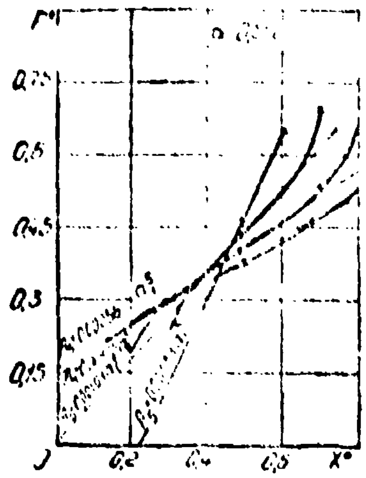

Fig. 5. Smoothed functions of the shift in the nonlinear element for the slowly changing component in the slip mode.

In order to determine the slowly changing component $F^{o}(x^{o})$, in equation (11) it is necessary to exclude $\bar{A}$, for which we plot the dependence of the amplitude of pulsations $\bar{A}$ on the shift in x^{o} when A_5 = const, having used the graph depicted on Fig. 4, taking conditions (9) and (10) into account. Then, according to formula (11), exluding $\bar{A}$, we plot the dependence $F^{o}(x^{o})$. The found functions of the shift in the nonlinear element in the slip mode (when $\alpha = 0.5\pi$) are given on Fig. 5. After the linearization, the obtained characteristics are described by equations (when $\alpha = 0.5\pi$)

$$\begin{aligned}
F^{o} &= 0{,}45x^{o} + 0{,}156 \quad \text{при } A_5 = 0{,}0025a_1^5; \\
F^{o} &= 0{,}536x^{o} + 0{,}138 \quad \text{при } A_5 = 0{,}00175a_1^5; \\
F^{o} &= 0{,}875x^{o} + 0{,}006 \quad \text{при } A_5 = 0{,}0010a_1^5; \\
F^{o} &= 1{,}68x^{o} - 0{,}358 \quad \text{при } A_5 = 0{,}0005a_1^5.
\end{aligned} \tag{19}$$

Figure 6 gives dependences of the amplification factor $\frac{A_5}{a_1^5}$, calculated by the described method, on the curvature of the linearized characteristic of the nonlinear element k for the slowly changing component for different switching angles of the structure α.

By using the obtained dependences (19) and substituting them into the first equation of system (14), it is possible to construct by the method of trapezoidal frequency characteristics the transient process for the slowly changing component in the slip mode both for the controlling and disturbing actions. The curve of the transient process for the slowly changing component for the controlling action when $\alpha = 0.2\pi$ and $A_5 = 0.001\ _1^5$, plotted by the method of trapesoidal frequency characteristics, is given on Fig. 7. Curves of the

transient process for different values of the amplification factor A_5 can be similarly plotted.

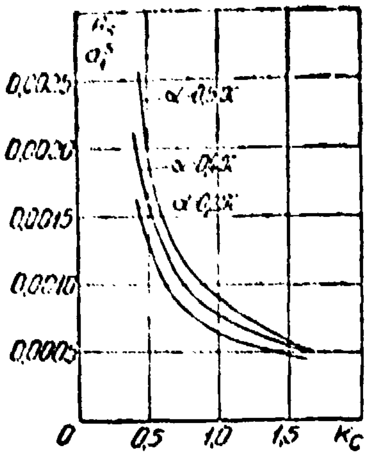

Fig. 6. Dependence of the amplification factor of the system on the curvature of the linearized characteristic of the nonlinear element.

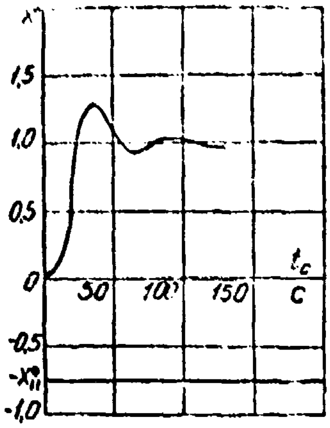

Fig. 7. Transient process of the TsSS SPS according to the controlling action. Here $t = \frac{t_n}{a_1}$, where t is the time.

Results of theoretical studies of slip modes of the TsSS SPS were conducted on an electrodynamic model.

Figure 8 gives an oscillogram of the section of the change in rate from V = 10 m/s to V = 0 with constant acceleration for selected values of parameters $\alpha = 0.2\pi$, $A_5 = 0.001a_1^5$ and $a_1 = 42.14\ s^{-1}$. The calculation of parameters of the model is carried out according to the method described in [6]. Operation of the TsSS SPS was conducted in the slip mode with a variable quantization increment . The criterion of the coincidence of results of theoretical and experimental studies is the magnitude of error [mismatch] of the path, which, according to the calculations, must not exceed the quantization increment for any value of the rate. The slip mode, which

consists in oscillations in the error of the path by a magnitude of one quantum is clearly marked on the oscillogram (Fig. 8). Owing to the fact that the traveling quantum is selected as variable and increasing with an increase in the rate, the frequency of the oscillations is changed insignificantly with a change in the rate from maximal to zero, Evident are pulsations of the slowly changing component, which consist in pulsations of rate. The error with respect to rate is such that the magnitude of the error of the path does not exceed the quantization increment, which indicates the high-speed operation of the system of automatic control.

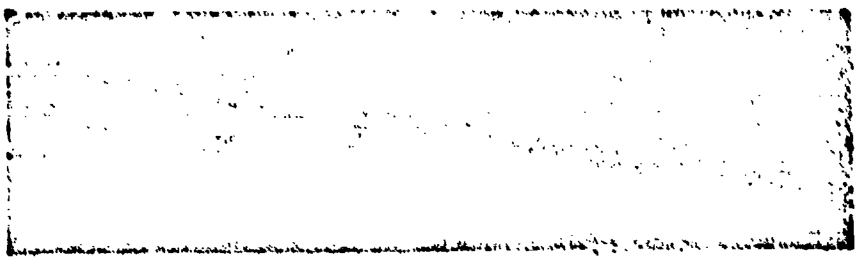

Fig. 8. Oscillogram of the section of slowing down of the TsSS SPS in the slip mode: 1 - assigned rate of movement; 2 - actual rate of movement; 3 - current; 4 - error of the path.

The proposed method makes it possible, with an accuracy sufficient for practical calculations, to calculate parameters of the slip mode in the digital tracking control system with variable structure, the dynamics of which is described by the nonlinear differential equation of a high order.

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